Written Exam for the M.Sc. in Economics Winter 2011/2012

ADVANCED MACROECONOMETRICS

Final Exam
January 25, 10:00 – January 27, 10:00

PLEASE NOTE that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as one PDF document (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

FOCUS ON EXAM CHEATING: In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Practical Information

Note the following formal requirements:

- This is an *individual* examination. You are not allowed to cooperate with other students or other people, see the *focus on exam cheating* above.
- The assignment consists of Sections 1-7 with 21 questions to be answered. *Please answer all questions.*
- The exam paper should not exceed 20 pages. A maximum of 20 pages of supporting material (graphs, estimation output, etc.) can accompany the paper as appendices. You may refer to the computer output in the appendices when answering the questions. Also, you may add clarifying comments in the output as part of your answer.
- All pages must be numbered consecutively and marked with your *exam number*. You should *not* write your name on the exam paper.
- *Your paper must be uploaded on the course page in Absalon at the given time.* The exam paper (including supporting material) must be in *PDF-format* and collected in *one file only*; the uploaded file must be named `1234.pdf`, where `1234` is your exam number.

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

1 Background

This project examination deals with econometric models for international parities such as the purchasing power parity (PPP), and the uncovered interest rate parity (UIP). The purpose of the assignment is to assess your ability to use statistical procedures to make inference on the equilibrium structures and the dynamic adjustment properties, as well as your ability to interpret the results.

The data set you are given consists of the five variables

P_Dom	:	Price level for the domestic economy ($2005 = 1$).
P_For	:	Price level for the foreign economy ($2005 = 1$).
Exch	:	Exchange rate denominated as domestic currency per unit of the foreign currency ($2005 = 1$).
R_Dom	:	Domestic interest rate (bond rate in percent *p.a.*).
R_For	:	Foreign interest rate (bond rate in percent *p.a.*).

All variables are observed monthly from $1994:1$ to $2010:12$. For the empirical analysis, define the following transformed variables

$$\begin{aligned} p_t &= \log(\mathsf{P\_Dom}_t) \\ p_t^* &= \log(\mathsf{P\_For}_t) \\ p_t - p_t^* &= \log(\mathsf{P\_Dom}_t/\mathsf{P\_For}_t) \\ s_t &= \log(\mathsf{Exch}_t) \\ \Delta p_t &= p_t - p_{t-1} \\ \Delta p_t^* &= p_t^* - p_{t-1}^* \\ R_t &= \mathsf{R\_Dom}_t/1200 \\ R_t^* &= \mathsf{R\_For}_t/1200 \end{aligned}$$

Here p_t and p_t^* are natural logs of the price levels, $p_t - p_t^*$ denotes the log of relative prices, s_t denotes the log of the nominal exchange rate, Δp_t and Δp_t^* denote the monthly inflation rates in the domestic and foreign economy, respectively, while R_t and R_t^* denote the monthly interest rates in fractions and not in percentages. Most of the empirical analysis considers the $p = 5$ dimensional data vector

$$x_t = (p_t - p_t^* : s_t : R_t : R_t^* : \Delta p_t)'.$$

One famous theoretical parity condition relating the variables in equilibrium would be the PPP, stating that due to goods arbitrage relative prices measured in the same currency should be stable, i.e.

$$p_t - p_t^* - s_t = u_{1t}, \tag{1.1}$$

where u_{1t} is a stationary process, $u_{1t} \sim I(0)$. Another possible candidate for an equilibrium relationship could be the *ex post* UIP condition stating

$$R_t = R_t^* + \Delta s_t + u_{2t}, \tag{1.2}$$

with $u_{2t} \sim I(0)$. Other candidates may include stable *ex post* real interest rates

$$R_t - \Delta p_t = u_{3t} \tag{1.3}$$

$$R_t^* - \Delta p_t^* = u_{4t}, \tag{1.4}$$

or the real interest rate parity

$$R_t - \Delta p_t = R_t^* - \Delta p_t^* + u_{5t}, \tag{1.5}$$

with $u_{3t}, u_{4t}, u_{5t} \sim I(0)$.

This assignment guides you through a cointegration analysis of the variables in x_t. In the analysis of the long-run structure we consider (1.1), (1.2), (1.3), (1.4), and (1.5) as *theoretical* candidates for cointegrating relationships. The candidates above are shown without deterministic terms, but in practice, deterministic variables may be needed to balance the autonomous growth in the processes or to take account of special events within the considered sample.

As a background for the analysis, you are informed that the foreign economy has a very strong seasonal variation in production due to large agricultural and mining sectors. You are also informed that the statistical offices in the two countries changed the compilation of price indices during the year 2006, and that the effects on the published statistics are still unknown.

[1] Construct the relevant variables for the empirical modelling, x_t, and perform a graphical analysis of the time series. Try, in particular to tentatively judge the degree of integration of the variables p_t, p_t^*, $p_t - p_t^*$, s_t, Δp_t, R_t, and R_t^*. Also try to assess whether the theoretical candidates for equilibrium relationships above are empirically relevant.

[2] Based on your conclusions, specify a theoretically and empirically relevant scenario, i.e. a consistent impression of the number of stochastic trends and their loadings, and the cointegration properties.

2 The Statistical Model

Consider the $p-$dimensional vector autoregression:

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + ... + \Pi_k x_{t-k} + \phi D_t + \epsilon_t, \tag{2.1}$$

for $t = 1, 2, ..., T$ with initial values, $x_{-k+1}, ..., x_{-1}, x_0$, and where the error term is assumed to be independently Gaussian distributed, $\epsilon_t \sim N(0, \Omega)$. The vector D_t contains potential deterministic variables, such as a constant, a trend, and dummy variables relevant for the empirical analysis.

[3] Construct and estimate a well-specified statistical model for x_t similar to (2.1). Outline how you proceed and argue for your choice of the number of autoregressive lags, k, and the the relevant specification of deterministic terms.

[4] State the maintained assumptions for the statistical model and test that they are fulfilled for your empirical model. In practice it may not be possible to find a model that is acceptable in all directions, just do as well as you can.

3 The Cointegration Rank

[5] For your preferred model above, derive the corresponding vector error correction form (VECM):

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-(k-1)} + \phi D_t + \epsilon_t.$$

[6] Explain how the likelihood ratio test statistics for the reduced rank of Π are calculated.

[7] Explain the concept of *similarity* of the rank test procedure, and explain how this is important for the preferred way to include the deterministic components in the model.

[8] Explain how the asymptotic distribution of the rank test statistic, involving Brownian motions, can be simulated using random walks.

[9] Determine the cointegration rank, $r = \text{Rank}(\Pi)$, for your preferred model. Discuss if the different sources of information on the cointegration rank point in the same direction or if you find conflicting evidence.

4 Testing Hypothesis

[10] Impose the reduced rank, $\Pi = \alpha\beta'$, and estimate the cointegrated VAR (CVAR) model. Comment on the results. Explain what it means that the individual parameters are not identified.

[11] Test for *long-run exclusion* for all variables in the model–including potential deterministic terms, i.e. the hypothesis that a particular variables does not enter the long-run relationships.
Explain how to calculate the degrees of freedom.
Comment on the implications for the theoretical candidates in Section 1.

[12] Next, test for the *stationarity* of the variables in x_t.
Explain again how to calculate the degrees of freedom.
Comment on the implications for the theoretical candidates in Section 1.

[13] Finally, test whether each of the theoretical candidates for cointegrating relationships, (1.1), (1.2), (1.3), (1.4), and (1.5) may be considered stationary in your data–both with and without allowance for additional deterministic terms.
For each accepted case discuss whether the implied error correction is in line with what you would expect from economic theory.

5 Identification

[14] Consider a restricted cointegration space

$$\beta = (\beta_1 : \beta_2 : ... : \beta_r) = (H_1\varphi_1 : H_2\varphi_2 : ... : H_r\varphi_r)\,.$$

State and explain the conditions under which the structure is *generically* identifying the cointegrating relationships.

Explain what it means that a structure is also *empirically* identified.

[15] Derive an identified structure for the empirical model.

Discuss–in detail–the results in terms of the significance of the estimated parameters and the interpretability in light of economic theory.

[16] Explain the idea of recursive estimation.

Explain why the recursively estimated eigenvalues are informative on the stability of the error correction coefficients in α.

Perform a recursive estimation of the identified structure and discuss the results.

6 The Moving Average Representation

[17] Estimate and interpret the Granger representation for your preferred identified structure and compare with the scenario in question [2].

Explain in detail how the included deterministic variables are propagated through the autoregressive structure and how they ultimately affect the variables in levels, x_t, and the deviations from the cointegrating relationships, $\beta' x_t$.

[18] Now consider the hypothesis that one of the stochastic trends is generated by cumulated innovations to the exchange rate,

$$ST_t^a = \sum_{i=1}^{t} \epsilon_{s,i},$$

where $\epsilon_{s,t}$ is the error term in the equation for Δs_t. Explain how this hypothesis would change the Granger representation, and explain how the hypothesis can be tested.

[19] Next consider the hypothesis that one of the stochastic trends is generated by cumulated innovations to the real domestic interest rate,

$$ST_t^b = \sum_{i=1}^{t} (\epsilon_{R,i} - \epsilon_{\Delta p,i})\,,$$

where $\epsilon_{R,t}$ and $\epsilon_{\Delta p,t}$ are the error terms in the equations for ΔR_t and $\Delta^2 p_t$, respectively. Explain how this hypothesis would change the Granger representation, and explain how the hypothesis can be tested.

7 Extensions

[20] (Robustness) In some situations, the obtained results from an empirical model are very sensitive to specific choices in the modelling process, e.g. the chosen specification of dummy variables, potentially included level shifts, the choice of cointegration rank, etc.

For the choice that you where most in doubt with in the modelling process above, perform a robustness analysis to see if the results depend critically on the choice you made.

[21] (I(2)–Model) The I(1) data vector considered so far,

$$x_t = (p_t - p_t^* : s_t : R_t : R_t^* : \Delta p_t)' ,$$

may be considered as a transformation of a data set in levels

$$y_t = (p_t : p_t^* : s_t : R_t : R_t^*)' .$$

Assume that the nominal price indices, p_t and p_t^*, are integrated of second order, I(2), and that s_t, R_t, and R_t^* are integrated of order one, I(1), and consider a tentative VAR model for y_t.

Discuss the condition on the model that would allow a valid *nominal-to-real transformation* from the I(2) data y_t to the I(1) data x_t and that would be consistent with your findings above.

Explain which deterministic variables you would allow in the model for y_t if you were asked to estimate the I(2) model and test the nominal-to-real transformation.